Class: BSM-S5

Subject: **Mathematical Statistics**

Total Time Allowed: 3 Hours

Name: _____

Date: _____

Instructor: Hina Naz

Max Marks: 50

Registration #_____

Question no 1: Marks: 10

(a) What are the conditions required for a random variable to have a hypergeometric distribution?

(b) A really bad carton of 18 eggs contains 5 spoiled eggs. An unsuspecting chef picks an eggs at random for his "Mega-Omelet Surprise". Find the probability that the number of unspoiled eggs among the 4 selected is **(i)** exactly 4 **(ii)** 2 or fewer **(iii)** more than 1

(c) A razor blade manufacturing company has a chance of 1 defective blade in 500 blades. The blades are marketed in packets of 5 blades. One hundred packets are supplied to a retailer. Find the number of packets which are likely to have no, 1 or 2 defective blades. Also find mean and variance.

Question no 2: Marks: 10

(a) Past experience reveals that Mr Rana can hit the Target 3 times out of 5 shots. Find the probability of hitting the target, **(i)** 4 times out of 6 shots **(ii)** at least 2 times out of six shots **(iii)** not more than once out of six shots

(b) Find Least-Squares Estimates a and b in Simple Linear Regression.

Question no 3: Marks: 10

(a) The time taken by the milkman to deliver to the high street is normally distributed with a mean of 12 minutes and a standard deviation of 2 minutes. He delivers milk every day. Estimate the number of days during the year when he takes **(i)** longer than 17 minutes, **(ii)** less than ten minutes, **(iii)** between nine and 13 minutes.

(b) For the standard normal distribution, find the area within one and half standard deviation of the mean – that is the area between $\mu - 2.3\sigma$ and $\mu + 2.3\sigma$.

(c) Find the probabilities for the standard normal distribution **(i)** $P(-1.56 \le z \le 2.31)$, **(ii)** $P(z > -.75)$

Question no 4: Marks: 10

(a) A paint manufacturing company claims that the mean drying time for its paints is not longer than 45 minutes. A random sample of 20 gallons of paints selected from the production line of this company showed that the mean drying time for this sample is 49.50 minutes with a standard

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deviation of 3 minutes. Assume that the drying times for these paints have a normal distribution (i)Using the 1% significance level, would you conclude that the company's claim is true.(ii) what is the type I error in this example? Explain in words. What is the probability of making such an error?

(b) A researcher believes that the mean age of medical doctors in a large hospital system is older than the average age of doctors in the United States, which is 46. Assume the population standard deviation is 4.2 years. A random sample of 30 doctors from the system is selected, and the mean age of the sample is 48.6. Test the claim at $\alpha = 0.05$.

Question no 5:   Marks: 10

A diabetic is interested to determining how the amount of aerobic exercise impacts his blood sugar. When his blood sugar reaches 170 mg/dL, he goes out for a run at a pace of 10 minutes per mile. On different days, he runs different distance and measures his blood sugar after completing his run. Note: the preferred blood sugar level is in the range of 80 t0 120 mg/dL. Levels that are too low or too high are extremely dangerous. The data generated are given in the following table.

Percentage containing ads	37	43	58	49	70	28	65	32
Price ($)	5.50	6.95	4.95	5.75	3.95	8.25	5.50	6.75

i. Plot a scatter diagram for the above data. Does the diagram exhibit a linear relationship?

ii. Find the predictive regression equation of blood sugar level on the distance run. $Y_i = \alpha + \beta X_i + e_i$.

iii. Find the values of $\hat{Y}$ and show that $\Sigma(Y - \hat{Y}) = 0$.

iv. Show the errors by drawing vertical lines between scatter points and predicted regression line.

v. Find the standard deviation of random errors.

vi. Compute the coefficient of determination. What percentage of the variation in price is explained by the least squares regression of price on the percentage of magazine space containing ads? What percentage of this variation is not explained?