COMSATS University Islamabad (Vehari Campus)
DEPARTMENT OF Computer Sciences
Final Examination FA 18

Instructor: Dr. Asfand Fahad
Course: Multivariable Calculus (MTH-105)
Program: BSCS-B16, BSSE-B10
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Time: 3 Hours
Marks: 50
Date: 10-1-2019
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Note: Attempt all the questions and each question carries equal marks.

Q.1 (*i*) Let $x(t) = 4t^3 + 6t^2 - 72t$, $y(t) = 2t^3 + 12t^2 + 18t$ be parametric curve in plane. Find equation of all horizontal and vertical tangents to $(x(t), y(t))$.

(*ii*) Let $r(t)$ be a vector function with constant length. Show that the position vector $r(t)$ is orthogonal to its tangent vector.

(*iii*) Find the Tangent vector and the equation of Tangent line to the vector valued function $r(t) = 2t^3 i + (4 - \ln t)j$ at $t_0 = 1$.

Q.2 (*i*) Let

$$f(x,y) = \begin{cases} \frac{2x^2y}{x^5+x^2y}, & \text{if } (x,y) \neq (0,0), \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Is f continuous at $(0,0)$? Jutify your answer.

(*ii*) Let $f(x,y) = xy \sin xy + x^2 y e^{x+y^2}$. Find all second order partial derivatives of f.

Q.3 (*i*) Find the length of the curve produced by vector valued function $r(t) = e^t \cos t i + e^t \sin t j$ from $t = 0$ to $t = \pi$.

(*ii*) Evaluate $\int\int_R f(x,y)\, dA$, where $f(x,y) = 2x - y^2$ and R is the region enclosed between the $y = -x + 1$, $y = x + 1$ and $y = 4$.

Q.4 (*i*) Let $f(x,y) = x^3 + y^2 - 2xy + 7x - 8y + 4$. Locate all local extremum and saddle points and find local extremum values of $f(x,y)$.

(*ii*) Let $f(x,y) = x^2 + 3xy + y - 1$. Find the derivative of f in the directions of (*a*) $u = i$ and (*b*) $u = 3i - 4j$ at $(1,1)$.

Q.5 (*i*) Let $y = e^x$, where $0 \leq x \leq 1$. Find

(*a*) The area between the curve and the x-axis.

(*b*) Find the volume of the solid which is bounded above by $f(x,y) = xy$ and bounded below by the region described in part(*a*).

(*ii*) Write down the formula for the chain rule for functions of three variables and find the rate of change in $w = \cos xy$ along the parabola $x = -y^2$ at $(0,0)$.

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